Stress-Intensity Factor and Fatigue Crack Growth Analyses For Rotorcraft Using AGILE

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Abstract

To meet the challenging conditions involved in damage tolerance applications for rotorcraft components, a new and advanced software package has been developed. This technology, called Automated, Global, Intermediate, Local Evaluation (AGILE), was developed under the sponsorship of Federal Aviation Administration Rotorcraft Structural Integrity Program. In order to provide efficient and effective solutions for relatively small cracks in highly complex rotorcraft structures, AGILE uses a symmetric Galerkin boundary element method and a finite element method based alternating method to determine the fracture mechanics parameters that control fatigue crack growth. To assess the use, capability, functionality, and accuracy of the AGILE package, several generic structures with three-dimensional cracks under various loadings were analyzed. These analysis results are presented in this paper to demonstrate the capabilities of this unique new methodology as a necessary step towards establishing AGILE for rotorcraft applications.

1. Introduction

Rotorcraft airframe structures and dynamic components are much more complex than fixed-wing aircraft components, and, possibly excepting engines, they also accumulate cyclic loads that can lead to fatigue failures much more rapidly in practically every flight regime. With rapid accumulations of load cycles during operations, fatigue failures can result from miniscule manufacturing and service-induced defects. In addition, rotorcraft usage spectrums have also become more severe and diverse than fixed-wing aircraft. As a result, airworthiness assurance analyses for rotorcraft based on direct consideration of fatigue crack growth cannot always be adequately addressed by the current state-of-the-art damage tolerance (DT) analysis capabilities.

In 2000, to support Federal Aviation Administration (FAA) rulemaking and the development of certification guidance materials for a DT approach to the design and certification of rotorcraft airframe structures and dynamic components, the FAA William J. Hughes Technical Center established research and development (R&D) collaborations with the Rotorcraft Industry Technological Association. As a result of these collaborations, a Rotorcraft Damage Tolerance (RCDT) R&D Roadmap was developed to identify and prioritize ten areas in which research is needed to support FAA rulemaking. One of the high priority areas was the development of a fatigue crack growth computational methodology suited for RCDT applications. For this particular need, the FAA contracted Professor Satya Atluri to adapt and enhance his previous successful computational analysis modeling work for fixed-wing aircraft to rotorcraft.

More specifically, the FAA-sponsored research and development that was performed by the Center for Aerospace Research and Education at UCLA, and later at the University of California, Irvine (UCI), and Knowledge Systems Research (KSR), L.L.C. was specifically aimed at developing, validating, and transferring a computational mechanics analysis package for performing RCDT analyses. The approach that was taken in research at UCLA and UCI and development at KSR has led to a unique software package called Automated, Global, Intermediate, Local Evaluation (AGILE), a package that is aimed at helping rotorcraft structural designers meet certification requirements. AGILE has been tailored to cope with the propagation of very small, arbitrarily oriented, nonplanar cracks in complex rotorcraft materials and structures. The AGILE software can generally be used to model straight and curved cracks in both two-dimensional (2D) and three-dimensional (3D) bodies, to trace nonplanar crack growth, and to accommodate both elastic and elastic-plastic constitutive behavior.

Because of proprietary concerns leading to the general unavailability of public-released rotorcraft structures and loads data, the results provided in this paper are limited to generic bodies with 3D cracks under various loadings that were analyzed with a linear elastic version of AGILE 3D. The analyses provide stress-intensity factor solutions and fatigue crack growth life under spectrum loadings for five specific crack and structure geometries. The results that are presented in this paper are intended to demonstrate the use, capability, functionality and efficiency of the AGILE software package as a first step towards actual rotorcraft applications.

2. AGILE Background

AGILE is a robust fracture mechanics analysis code for the analyses of multiple curved cracks in a thin sheet or arbitrary nonplanar cracks in a solid body. AGILE uses a symmetric Galerkin boundary element method (SGBEM) and a finite element method (FEM) based alternating method to determine the fracture mechanics parameters that control fatigue crack growth. AGILE includes three subpackages: a 2D solver, a 3D solver, a graphical user interface (GUI), with load/boundary condition transferors. The AGILE GUI is developed on MSC.PATRAN, which supports major commercial FEM codes, including MSC.NASTRAN and ANSYS. A complete description of the technology underlying the AGILE software is beyond the scope of this paper. The interested reader can find detailed information on the finite element alternating method (FEAM) in references 1-3 and on the SGBEM in references 4-5.

2.1 SGBEM-FEM Alternating Method Iteration Procedure

As described in the referenced sources, the basic steps of the SGBEM-FEM alternating iteration procedure are: (i) using FEM, obtain the stresses at the location of the hypothetical crack in a finite uncracked body that is subjected to given boundary conditions; (ii) using SGBEM, solve the problem of a crack, the faces of which are subjected to the tractions found from FEM analysis of the uncracked body; (iii) determine the residual forces at locations corresponding to the outer boundaries of the finite uncracked body that result from the displacement discontinuities at the crack surface; (iv) using FEM, solve a problem for a finite uncracked body under residual forces from SGBEM analysis; and (v) obtain the stresses at the location of the crack corresponding to FEM solution. Steps (ii) to (v) are repeated until the residual load is sufficiently small. Usually, less than ten iterations are enough for convergence. Then, by summing all the appropriate contributions, the total solution for a finite body with the crack is obtained. Finally, having the converged solution, the stress-intensity factors for each of the modes can be calculated from the near tip crack opening displacements.

2.2 Crack Growth Procedure

To model fatigue crack growth it is only necessary to add another element layer to the existing crack model. To advance a point at the front of a nonplanar crack, it is necessary to know the direction and extent of crack growth. The formulation given by Cherepanov [6] has been found to provide the most effective criterion. In this formulation, crack growth occurs in the direction of the vector $\Delta\vec{\mathbf{K}}$ with the crack growth rate determined by the relative magnitude of $\Delta\vec{\mathbf{K}}$ using a conventional fatigue crack growth relationship (e.g., NASGRO, Walker, and Paris crack growth equations); N.B., the vector $\Delta\vec{\mathbf{K}}$ is normal to the crack front.

The procedure for the advancement of the front of a nonplanar crack is (i) using the SGBEM-FEM alternating method, solve the problem for the current crack configuration and determine ranges for the stress-intensity factors for the nodes located at the crack front; (ii) for each node determine the local crack front coordinate system by its neighboring crack elements; (iii) for each node, calculate the crack advance Δa and the crack growth direction; (iv) move each node in the local crack front coordinate system and transform the movement to the global coordinate system. After terminating the crack growth procedure, the total number of cycles N is calculated as a sum of all the cycle increments.

2.3 Global-Intermediate-Local Hierarchical Approach

Analyses of complicated structures are performed by breaking down the analysis into a series of multiple smaller scale analyses. The first stage is denoted as the global stage, while the last stage is called the local stage. All others are referred to as intermediate stages. For example, a complicated problem can simply be broken down into a two-stage (global-local), a three-stage (global-intermediate-local), or even an n-stage (global-intermediate-….-intermediate-local) analysis. Hierarchical analyses broken down to more than three stages are possible, but will always have at least one global stage and one local stage.

The finite element (FE) model at each stage represents the entire or a portion of the entire structure. The global model begins by representing the entire structure, while the intermediate stages represent a subregion of the previous model, global or intermediate. The local model should cover a small portion of a single component of the structure that can be approximated by 2D plane or 3D solid elements of a single material. A boundary condition (BC) transfer is performed between each stage to solve for the unknown conditions along the boundary of the subregion of the current model. These displacement and loading conditions are extracted from the outputs of the previous model through a BC transfer process. Some BCs applied to the subregion, such as pressure may be predetermined from the initial problem specifications and can be directly applied using AGILE.

Conventional FE analysis is performed at each stage, except at the local one, wherein the FEAM is performed. The calculated outputs of the intermediate FE analysis are used to provide the appropriate BCs for the successive stage. Ultimately, FEAM analysis is performed on the local model, where fracture parameters such as the stress-intensity factors are calculated at each crack front node.

2.4 Graphical User Interface

The AGILE GUI has integrated AGILE's inherent fracture mechanics analysis capabilities into MSC.PATRAN. AGILE works within its mainframe forms, the menu bar, toolbar, command line, history list, and the graphics view-port. All user-defined commands pertaining to AGILE are recorded in the PATRAN session file and can be used to automatically regenerate the model. There are three major modules in this package: 2D and 3D FEM&SGBEM model creation and 2D Express FEM&SGBEM model creation. They convert the model data as well as results from the previous master FE models and create model files for AGILE 2D and 3D solvers.

3. Numerical Examples

3.1 Fatigue Crack Growth of a Semicircular Surface Crack

The first example considered is a semicircular surface crack in a plate as shown in Fig. 1. Uniform tensile stresses σ_0 applied at two opposite faces of the plate in the direction normal to the crack. Here, a is the radius of the semicircular crack. The plate and crack geometries considered are characterized by the geometric dimensions $h = 2''$; $w = 2''$; $t = 1''$; and $a = 0.4''$. The Poisson ratio $\nu = 0.3$ is chosen.

Solutions for this example problem have been provided in Han and Atluri [3]. The problem is a pure mode-I problem and has been solved by Raju and Newman [7] using the FEM, and by Frangi, et al. [8], using the SGBEM. The analytical solution is available for the infinite plate which shows lower K factors because the dimensions chosen for this problem are not large enough to represent a crack in the infinite plate.

As shown in Fig. 2, a comparison of the normalized stress-intensity factors by using the SGBEM-FEM alternating method with the referenced solutions shows a good agreement for all crack front locations. It is well known that the stress-intensity factors tend to zero in a boundary layer where the crack front approaches free surface of the body, when a surface crack breaks the outer surface at a right angle. This effect is also confirmed by using alternating method.

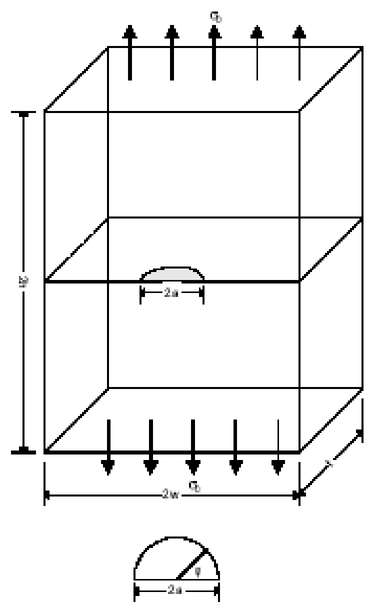

Figure 1 A Semicircular Crack in a Plate under Tension

The coarse mesh of crack surface and crack radius $a = 0.2''$ are used for fatigue crack growth analysis. The material for the plate is aluminum alloy 7075-T651. The NASGRO crack growth model was used for the crack growth analysis. This model is detailed in the reference manual of NASGARO 3.0 [9]. The material properties are listed in Table 1.

Constant and variable amplitude stress spectra were used for the crack growth analyses. The variable amplitude stress spectrum block ASTRIX is shown in Table 2. The maximum and minimum stresses corresponding to the three highest numbers of cycles in a step were used for the maximum and minimum stresses of constant-amplitude stress spectrum. Hence, the stress pair for the three constant-amplitude stress spectra are (19.2 ksi, 16 ksi), (18.4 ksi, 15.2 ksi), and (16 ksi, 12.8 ksi). The crack growth results are shown in Fig. 3. The fatigue crack growth lives are similar among the four cases of stress spectra.

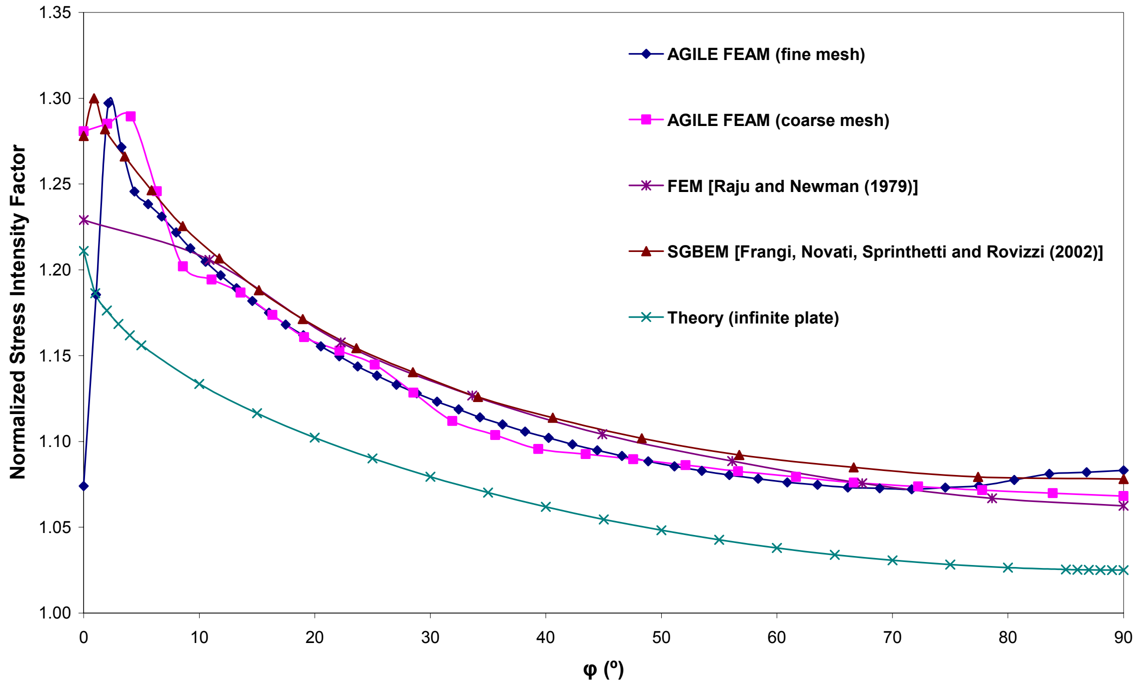

Figure 2 Normalized Stress-Intensity Factors ($K_I/2\sigma_0\sqrt{a/\pi}$) for a Semicircular Crack in a Plate

Table 1 Al 7075-T651 Material Properties

$C = 2.33\times10^{-8}$	$n = 2.885$
$p = 0.5$	$q = 1.0$
$K_{Ie} = 38$ ksi $\sqrt{\text{in}}$	$K_{IC} = 28$ ksi $\sqrt{\text{in}}$
$Kth = 3.0$ ksi $\sqrt{\text{in}}$	$Rcl = 0.7$
$C_{th}^{+} = 2.0$	$C_{th}^{-} = 0.1$
$\alpha = 1.9$	$S_{max}/\sigma_0 = 0.3$
$Ak = 1.0$	$Bk = 1.0$
$\sigma_{YS} = 76$ ksi	$\sigma_{UTS} = 85$ ksi
Thk = 1.0	$DK_0 = 3.0$

Table 2 ASTRIX Stress Spectrum

Step	No. of Cycles	S_{max} (ksi)	S_{min} (ksi)	R = S_{max}/ S_{min}
1	1222	20	16.8	0.840
2	2833	20	16	0.800
3	63976	19.2	16	0.833
4	7399	19.2	15.2	0.792
5	174009	18.4	15.2	0.826
6	6019	18.4	14.4	0.783
7	25542	17.6	14.4	0.818
8	1963	17.6	13.6	0.773
9	15169	16.8	13.6	0.810
10	75	16	13.6	0.850
11	51083	16	12.8	0.800
12	274	15.2	12.8	0.842
13	72	15.2	12	0.789
14	607	14.4	12	0.833
15	1948	14.4	11.2	0.778
16	4115	12.8	11.2	0.875
17	148	12.8	8.8	0.688
18	338	12	8.8	0.733
19	8	12	8	0.667
20	132	10.4	8	0.769
21	14400	10.4	7.2	0.692
22	132	9.6	7.2	0.750
23	8	9.6	5.6	0.583
24	8	8.8	4.8	0.545
25	131	8.8	-0.8	-0.091
26	8	6.4	-0.8	-0.125

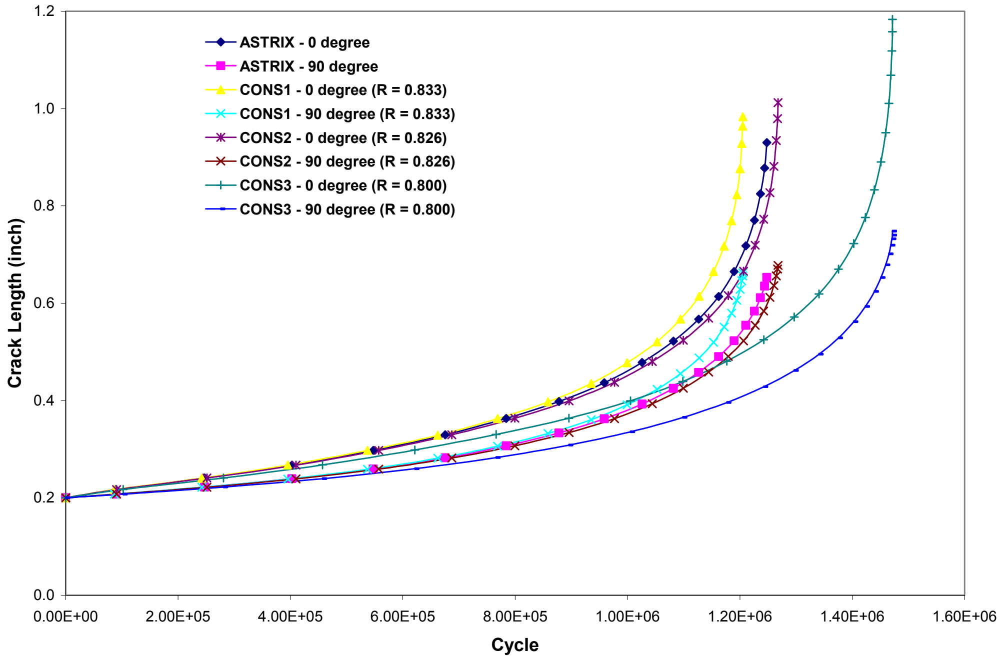

Figure 3 Crack Growth Curves Under Variable and Constant-Amplitude Stress Spectra for a Semicircular Crack in a Plate

3.2 Fatigue Crack Growth of a Corner Crack at a Circular Hole in a Finite Thickness Plate

The corner crack at a circular hole in a plate is considered and shown in Fig. 4. This example has been considered by many investigators for 3D fracture analyses with various methods. The geometry is characterized by the dimensions: $h = w = 24''$; $t = 1''$; $R = 1.5''$; a = 0.5″.
The Poisson ratio is taken as $\nu = 0.3$. Only half of the specimen was analyzed due to symmetry. The normalized stress-intensity factors along the crack front are plotted in Fig. 5. The results are compared to the available published solutions of Tan, Newman, and Bigelow [10] and Stress-Intensity Factors Handbook [11]. AGILE's stress-intensity factor solutions along the crack front are in general higher than the comparison solutions.

The crack radius $a = 0.2''$ are used for fatigue crack growth analysis. The material for the plate is aluminum alloy 7075-T651. Paris crack growth model was used for the crack growth analysis. The C and n property values used are listed in Table 1. Constant-amplitude stress spectrum with R = 0, S_{max} = 1 ksi was used. The crack length versus number of stress cycles results are shown in Fig. 6.

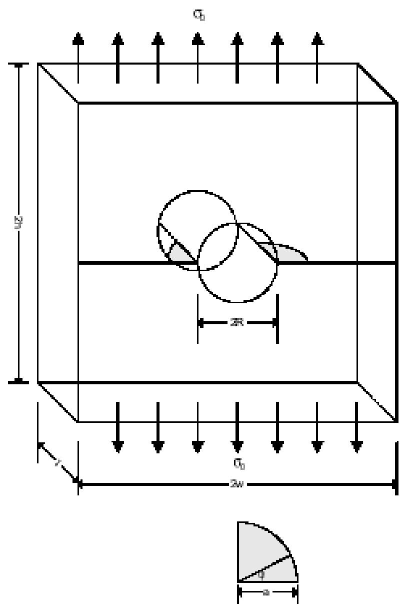

Figure 4 A Corner Crack at a Circular Hole in a Finite Thickness Plate Under Tension

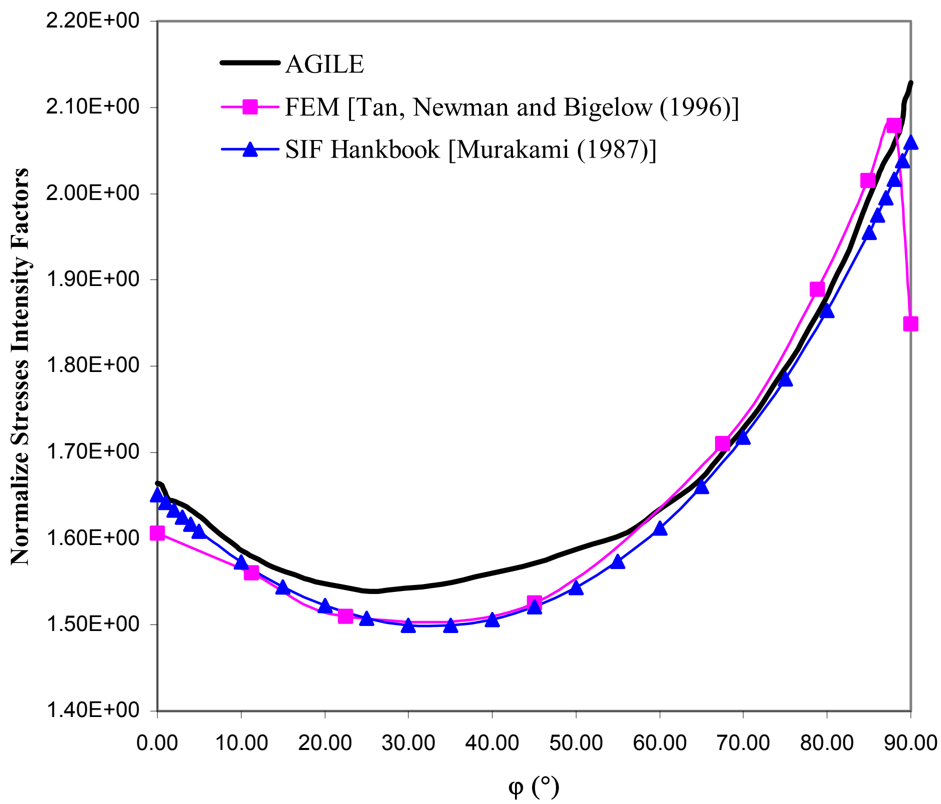

Figure 5 Normalized Stress-Intensity Factors ($K_I/\sigma_0\sqrt{a\pi}$) for a Corner Circular Crack at a Hole in a Finite Thickness Plate

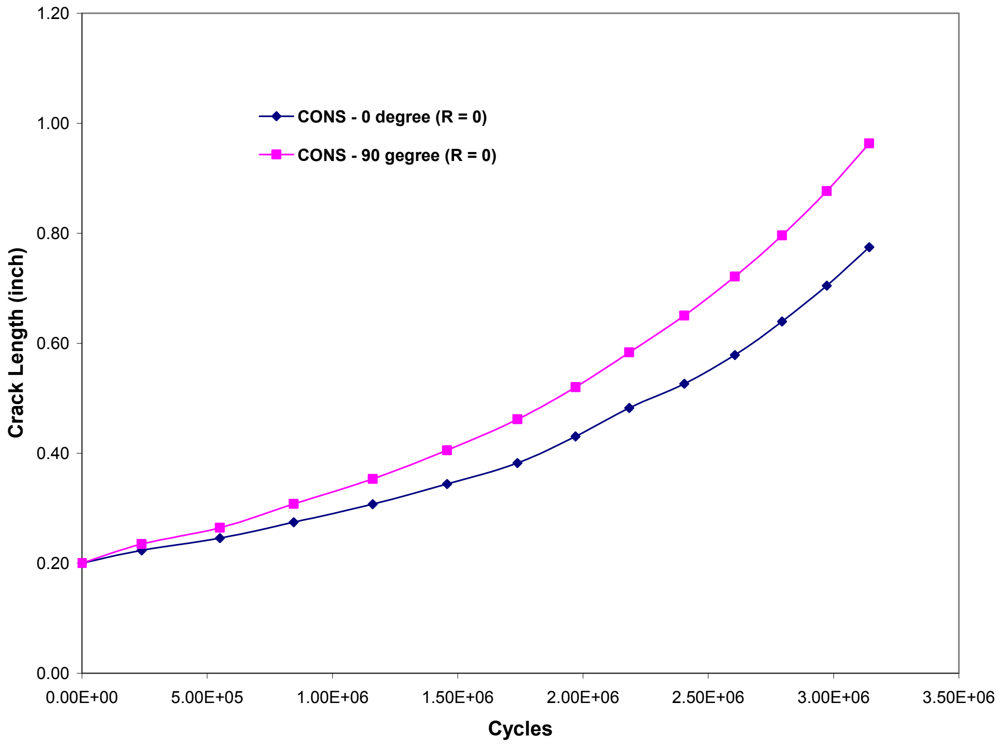

Figure 6 Crack Growth Curves Under Constant-Amplitude Stress Spectrum for a Corner Circular Crack at a Hole in a Finite Thickness Plate

3.3 A Semicircular Crack in a Thick Cantilever Cylindrical Tube Subjected to a Torque

A thick cantilever cylindrical tube (outside diameter = 4″, inside diameter = 2″, length = 24″) with a semicircular crack of 0.2″ radius located 14″ from the free end. The Poisson ratio is taken as $\nu = 0.3$. The applied load is a torque of 628 kip-in that was generated through linear-varying shear stress acting on the cross section in the circumferential direction at the free end of the tube. The shear stress at the inner edge of the tube is $\tau_0 = 25$ ksi. The meshes of the tube and the crack surface are shown in Fig. 7, while the normalized stress-intensity factors for modes I, II, and III are presented in Fig. 8. The mode I component is close to zero in this load case.

(a) (b)

Figure 7 Meshes for a Semicircular Crack in a Thick Cantilever Cylindrical Tube

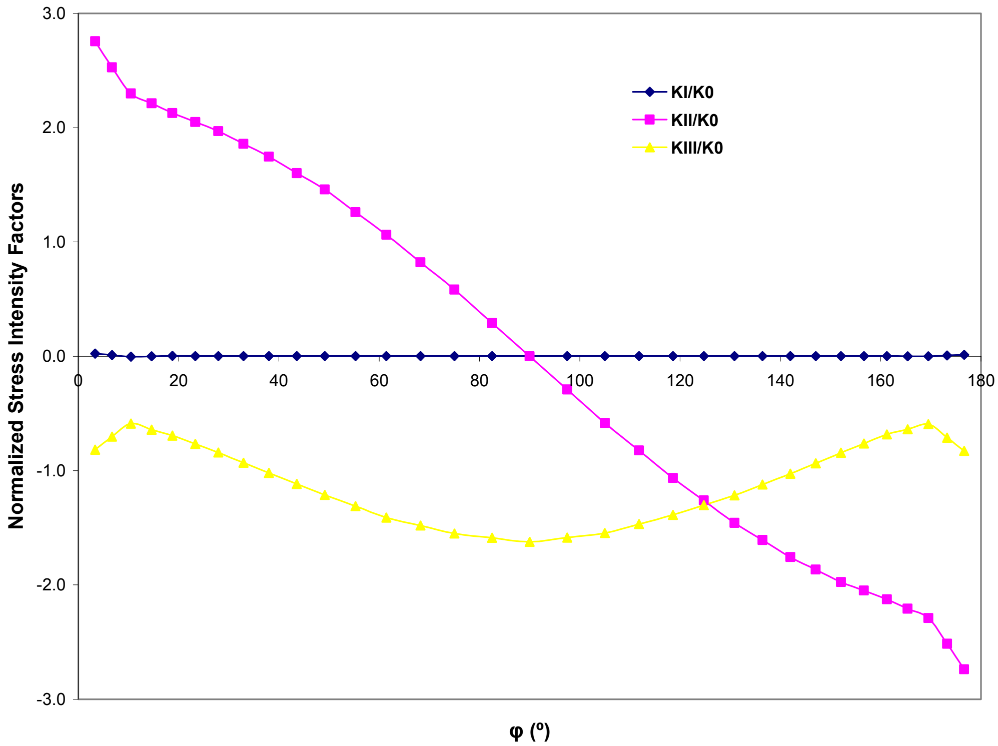

Figure 8 Normalized Stress-Intensity Factors (K_I/K_0, K_{II}/K_0, and K_{III}/K_0; and $K_0 = 2\tau_0\sqrt{a/\pi}$) for a Semicircular Crack in a Cylindrical Tube Subjected to a Torque

3.4 A Quarter-Circular Corner Crack at the Edge of a Hole in a Plate with Multiple Holes

A cantilever plate with five circular holes subjected to a shear stress is shown in Fig. 9. The length (L) and width of the plate are 10″ and the thickness (h) is 0.5″. The radius of the hole is 0.2″. The Poisson ratio is taken as $\nu = 0.33$. The distributed load p in the downward direction acting along the free edge of the plate (opposite to the fixed edge), as shown in Fig. 9, is 0.5 kip/in. A quarter-circular corner crack with a radius of 0.2″ is located at the edge of the center hole, as depicted in Fig. 9.

Using the global-intermediate-local hierarchical approach, the original plate structure was analyzed using FEM. A local model of 2″ by 2″ was cut out from the center of the plate that contained the quarter-circular corner crack and the center hole. The loadings and boundary conditions were transferred to the local model from the global model analysis results. The local model with the transferred loadings and boundary conditions is shown in Fig. 10. The normalized stress-intensity factors of mode I, II, and III are shown in Fig. 11. The mode I, II, and III stress-intensity factors are normalized through the factor ($K_0 = p(L/h^2)\sqrt{a\pi}$). Mode I is the dominant mode for this load case.

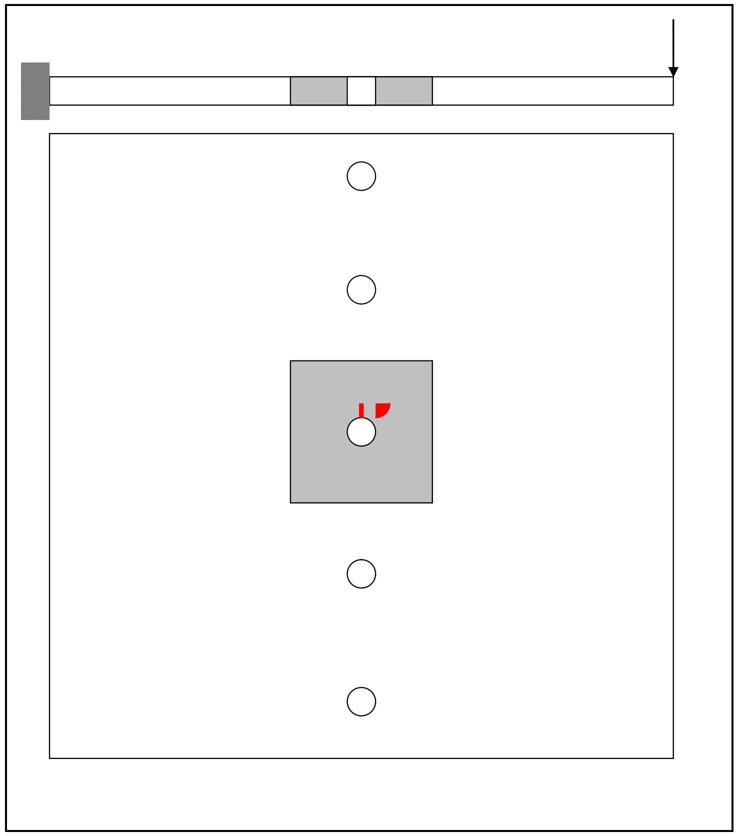

Figure 9 A Cantilever Plate with Five Circular Holes and a Quarter-Circular Corner Crack at the Edge of the Center Hole Subjected to a Shear Stress

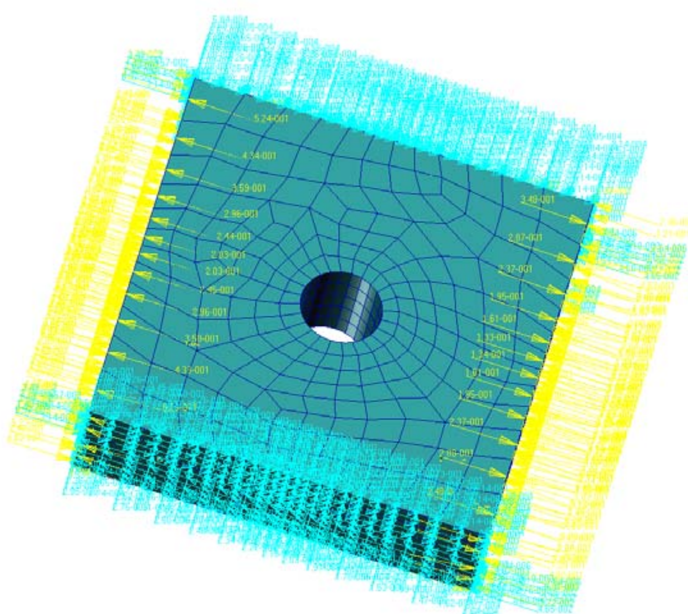

Figure 10 Local Model With the Transferred Loadings and Boundary Conditions

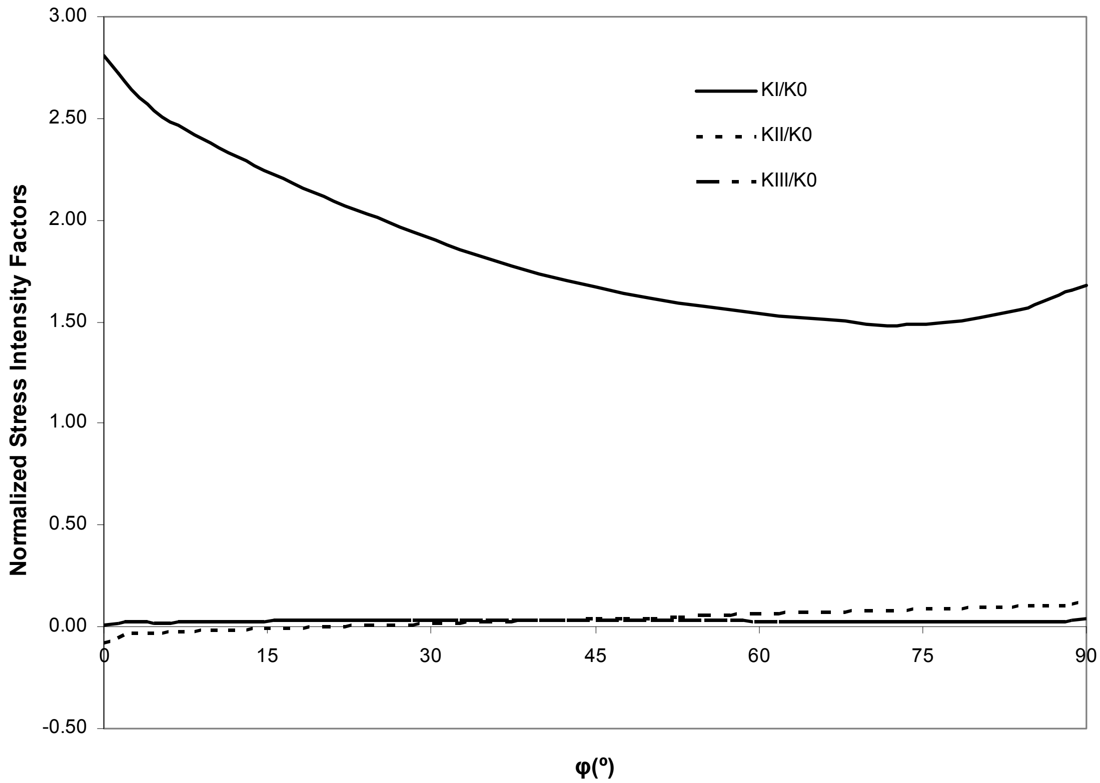

Figure 11 Normalized Stress-Intensity Factors for a Quarter-Circular Corner Crack at the Edge of a Circular Hole in a Plate Under Shear Stress

3.5 Nonplanar Fatigue Crack Growth of an Inclined Semicircular Surface Crack in a Test Specimen

Fatigue growth of an inclined surface crack in a plate is considered. As shown in Fig. 12, the modified ASTM E 740 specimen has been tested for the mixed-mode fatigue growth by Forth, et al. [12]. The specimens were taken from actual parts made from 7075-T73 aluminum. The crack orientation $\varphi = 30^{\circ}$ is used. Maximum tensile stresses $\sigma_0 = 15.88$ ksi are applied with a load ratio $R = 0.7$. The NASGRO fatigue crack growth model is used [9]. The material constants are taken as listed in Table 3.

The AGILE crack length versus stress cycles results are compared with the test data of four specimens as shown in Fig. 13. AGILE grew the crack from 0.05″ until the specimen was broken, automatically without any human intervention. The critical depth of the crack was 0.29″, while the experimental specimens report 0.34″, 0.23″, 0.32″, and 0.25″, with an average as 0.284″. AGILE’s prediction on crack growth life was good compared with the test data.

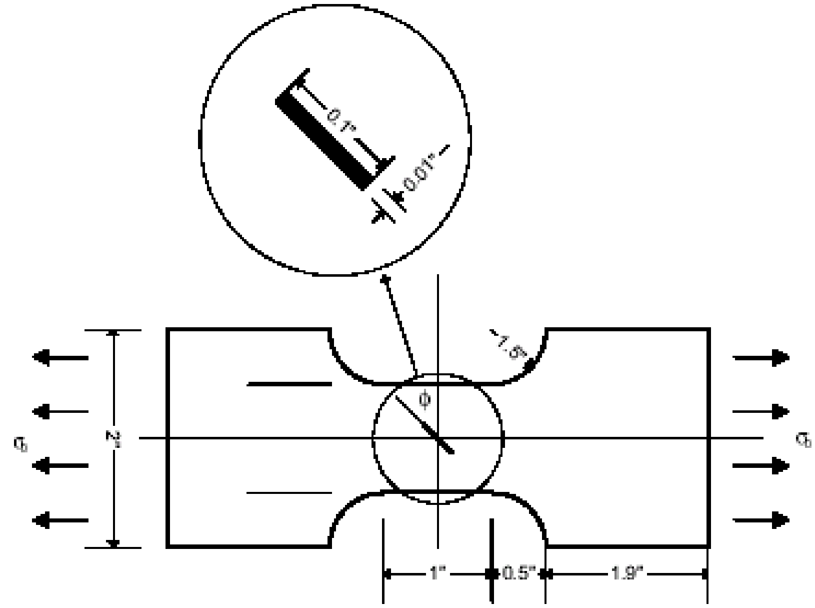

Figure 12 An Inclined Semicircular Surface Crack Specimen

Table 3 Al 7075-T73 Material Properties

$C = 1.49\times10^{-8}$	$n = 3.321$
$P = 0.5$	$q = 1.0$
$K_{Ie} = 50$ ksi√in	$K_{IC} = 28$ ksi√in
$Kth = 3.0$ ksi√in	$Rcl = 0.7$
$C_{th}^{+} = 2.0$	$C_{th}^{-} = 0.1$
$\alpha = 1.9$	$S_{max}/\sigma_0 = 0.3$
$Ak = 1.0$	$Bk = 1.0$
$\sigma_{YS} = 60$ ksi	$\sigma_{UTS} = 74$ ksi
Thk = 1.0	$DK_0 = 3.0$

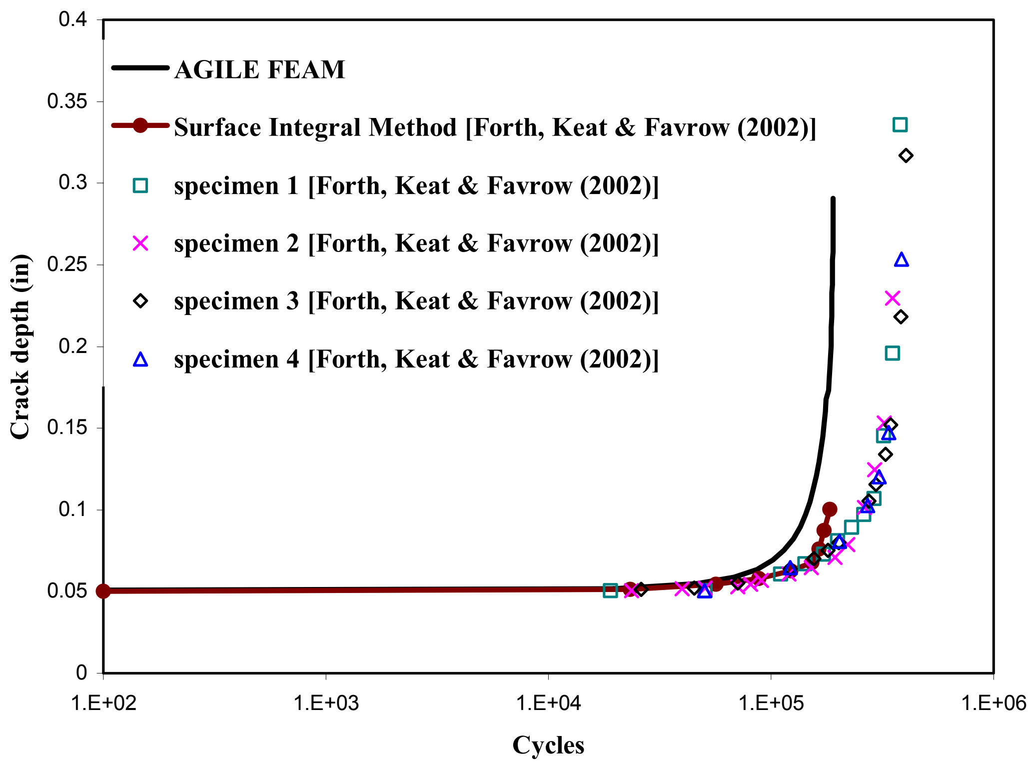

Figure 13 Fatigue Crack Growth of an Inclined Semicircular Surface Crack in a Tensile Test Specimen

4. Summary and Conclusions

The challenging conditions that are associated with rotorcraft damage tolerance (RCDT) have led to a comprehensive software package, Automated, Global, Intermediate, Local Evaluation (AGILE), for fracture mechanics and fatigue crack growth determinations. While direct applications of AGILE to actual rotorcraft components and flight conditions are underway, these have not yet been completed. This paper has provided an extensive set of analyses of generic examples of cracked bodies to illustrate the breadth of applications that AGILE can perform, and to judge its accuracy against known solutions. Specifically, five example analyses of generic structures with three-dimensional (3D) cracks under various loadings were analyzed by the use of AGILE 3D. The relatively good results of the stress-intensity factor solutions and the fatigue crack growth life under spectrum loadings has been demonstrated in this paper. The potential for the use, capability, functionality, and reliability of the AGILE software package in practical RCDT work is evident.

It can further be concluded that AGILE offers a substantial improvement in computational efficiency for RCDT analysis. Manpower costs can be dramatically reduced with AGILE's automatic crack growth features, which avoid mesh regeneration all together. The proficiency of the graphical user interface makes AGILE user-friendly and minimizes human errors typically associated with data preparation. The global-intermediate-local hierarchical approach reduces the problem scale and enables users to leverage the existing finite element models. Although validation and acceptance by the rotorcraft manufacturers are needed, it has been demonstrated that AGILE has the potential to become one of the tools in the damage tolerance design toolbox for rotorcraft industry.

5. Acknowledgement

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6. References

[1] Atluri, S. N., *Structural Integrity and Durability*, Tech Science Press, 880 pages, 1997.

[2] Nikishkov, G.P., Park, J.H., and Atluri, S. N., "SGBEM-FEM Alternating Method for Analyzing 3D Non-planar Cracks and Their Growth in Structural Components," *CMES: Computer Modeling in Engineering & Sciences*, Vol.2, pp.401-422, 2001.

[3] Han. Z. D. and Atluri, S. N., "SGBEM (for Cracked Local Sub-domain) – FEM (for uncracked global Structure) Alternating Method for Analyzing 3D Surface Cracks and Their Fatigue-Growth," *CMES: Computer Modeling in Engineering & Sciences*, Vol. 3, pp. 699-716, 2002.

[4] Bonnet, M., Maier, G., and Polizzotto, C., "Symmetric Galerkin Boundary Element Methods," *Applied Mechanics Reviews*, Vol. 51, pp.669-704, 1998.

[5] Han. Z. D. and Atluri, S. N., "On Simple Formulation of Weakly-Singular Traction & Displacement BIE, and Their Solutions through Petrov-Galerkin Approaches," *CMES: Computer Modeling in Engineering & Sciences*, Vol. 4, pp. 5-20, 2003.

[6] Cherepanov, G.P., *Mechanics of Brittle Fracture*, McGraw-Hill, New York, 1979.

[7] Raju, I. S. and Newman, J. C., Jr., "Stress-Intensity Factors for a Wide Range of Semi-Elliptical Surface Cracks in Finite-Thickness Plates," *Engineering Fracture Mechanics*, Vol. 11, pp. 817-829, 1979.

[8] Frangi, A., Novati, G., Springhetti, R., and Rovizzi, M., "3D Fracture Analysis by the Symmetric Galerkin BEM", *Computational Mechanics*, Vol. 28, pp. 220-232, 2002.

[9] Anon, *The Reference Manual of Fatigue Crack Growth Computer Program "NASGRO"*, Version 3.0, NASA Johnson Space Center, Report No. JSC-22267B, November 2001.

[10] Tan, P. W., Newman, J. C. Jr., and Bigelow, C. A., "Three-dimensional Finite-element Analyses of Corner Cracks at Stress Concentrations," *Engineering Fracture Mechanics*, Vol. 55, pp. 505-512, 1996.

[11] Murakami, Y., *Stress Intensity Factors Handbook*, Pergamon Press, 1987.

[12] Forth, S. C., Keat, W. D., and Favrow, L. H., "Experimental and Computational Investigation of Three-dimensional Mixed-mode Fatigue," *Fatigue and Fracture of Engineering Materials & Structures*, Vol. 25, pp 3-15, 2002.